# TREND: TempoRal Event and Node Dynamics for Graph Representation Learning

**Zhihao Wen**, Yuan Fang

School of Computing and Information Systems

Singapore Managent University

SMU SINGAPORE MANAGEMENT UNIVERSITY

School of Computing and Information Systems

# Outline

- ❑ **Introduction**
- ❑ Methodology
- ❑ Experiment
- ❑ Conclusion

# Issues of existing works

## 1. Take discrete snapshots

Dynamic network embedding: an extended approach for skip-gram based network embedding. IJCAI-2018.

DynGEM: Deep embedding method for dynamic graphs. arxiv-2018.

Dynamic network embedding by modeling triadic closure process. AAAI-2018.

Evolvegcn: Evolving graph convolutional networks for dynamic graphs. AAAI-2020.

## 2. Not inductive to new nodes

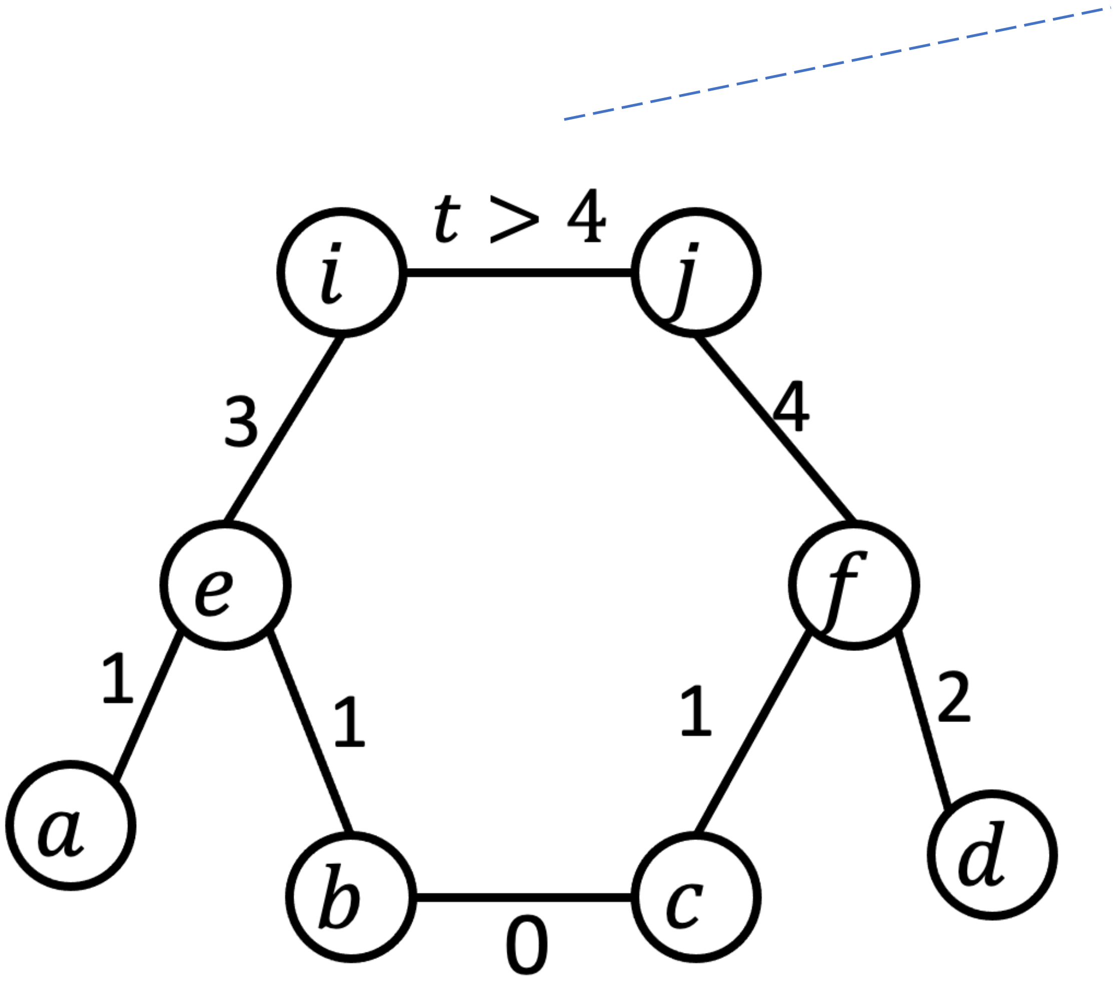
Embedding temporal network via neighborhood formation. KDD-2018.
Temporal network embedding with micro-and macro-dynamics. CIKM-2019.
Dynamic Heterogeneous Graph Embedding via Heterogeneous Hawkes Process. ECML-PKDD 2021.

## 3. Not model the exciting effects

DyRep: Learning representations over dynamic graphs. ICLR-2019.
Inductive representation learning on temporal graphs. ICLR-2020.
……

# Problem: temporal graph link prediction

Predict whether a **link** between ***i* & *j* at *t***



# Temporal point process and Hawkes Process

$$\lambda(t|\mathcal{H}(t)) = \lim_{\Delta t \to 0} \frac{\mathbb{E}[N(t+\Delta t)|\mathcal{H}_t]}{\Delta t}$$

Point process models **discrete sequential** events, assuming that **historical** events can influence the **current** event.

$$\lambda(t) = \mu(t) + \int_{-\infty}^{t} \kappa(t-s)\mathrm{d}N(s)$$

Hawkes process is desirable for modeling temporal link formation !
For current event is influenced **more** by **recent** events, **less** by **previous** events

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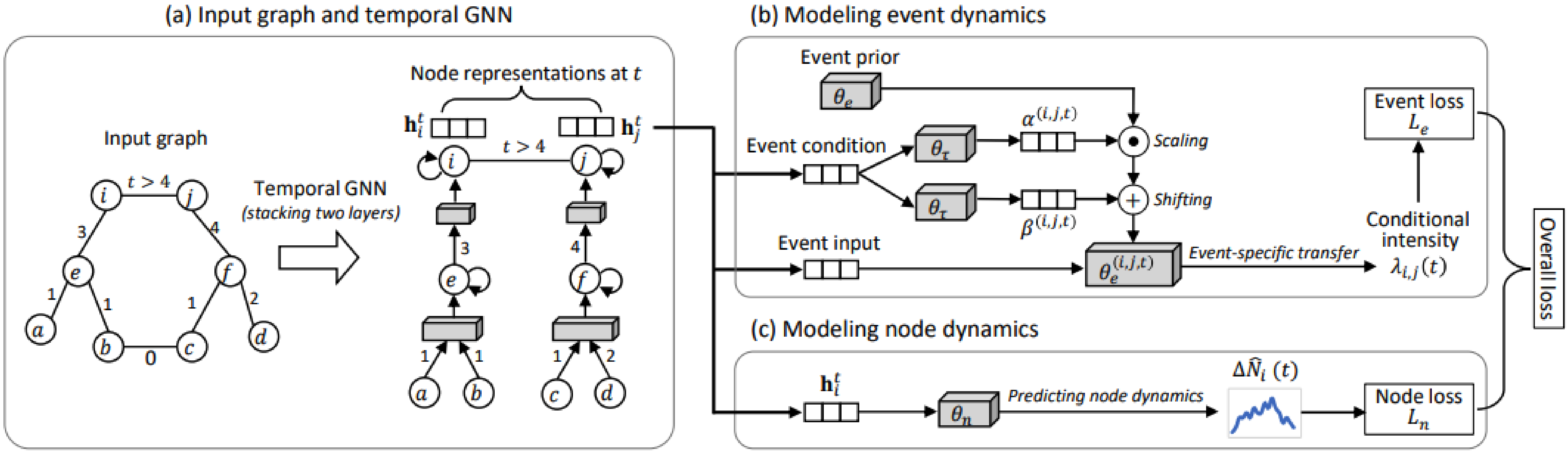
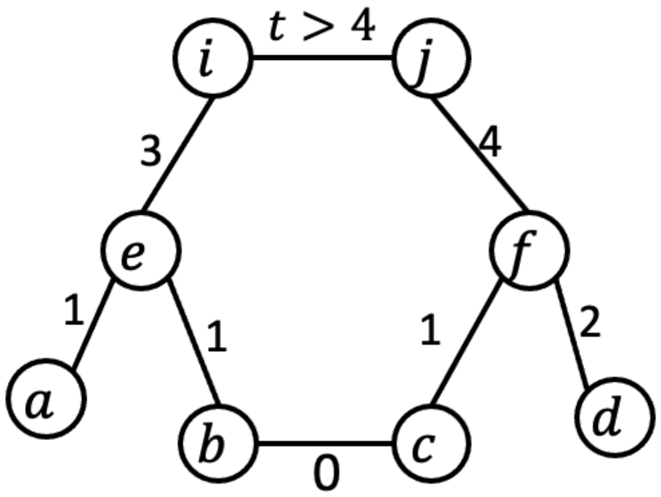
# Proposed model:TREND



# Hawkes Process on temporal graph



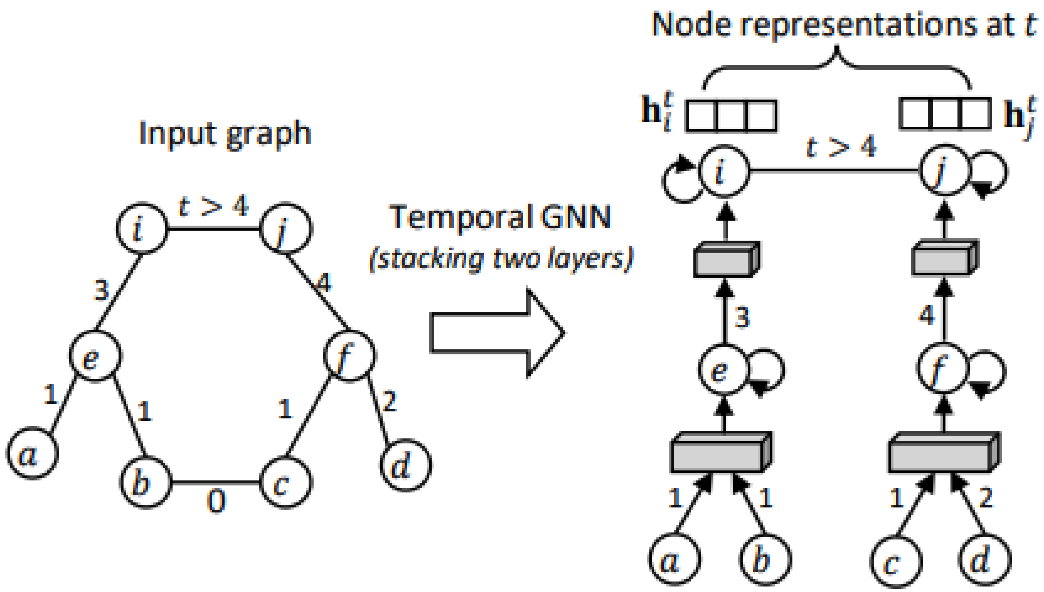

amount of excitement

$$\lambda_{i,j}(t) = \mu_{i,j}(t) + \sum_{(i,j',t')\in\mathcal{H}_i(t)} \gamma_{j'}(t')\kappa(t-t') + \sum_{(i',j,t')\in\mathcal{H}_j(t)} \gamma_{i'}(t')\kappa(t-t')$$

base rate

$\exp(-\delta(t-t'))$

# Hawkes Process on temporal graph

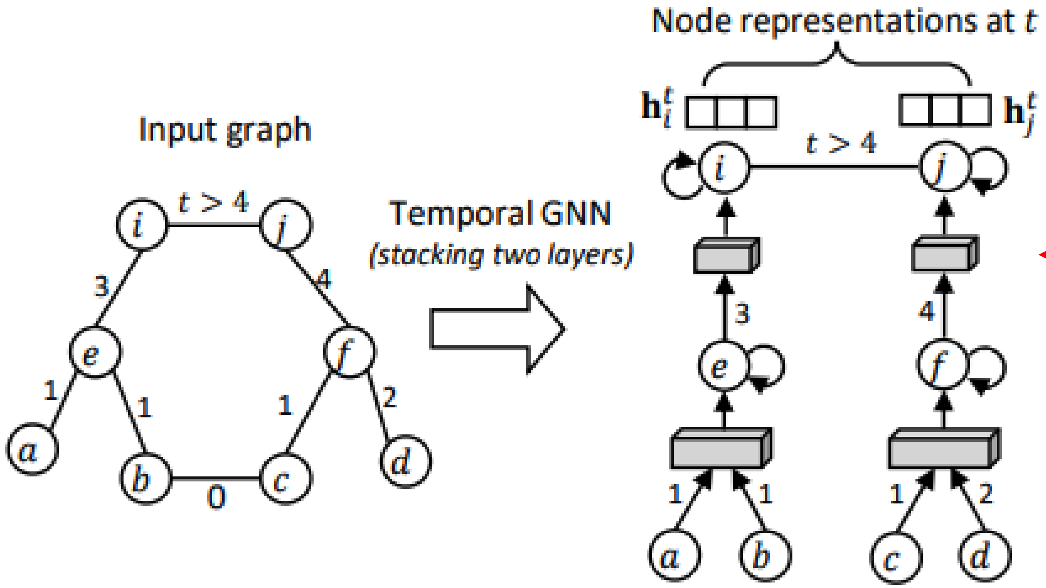



$$\lambda_{i,j}(t) = \mu_{i,j}(t) + \sum_{(i,j',t') \in \mathcal{H}_i(t)} \gamma_{j'}(t')\kappa(t-t') + \sum_{(i',j,t') \in \mathcal{H}_j(t)} \gamma_{i'}(t')\kappa(t-t')$$

↓

$$\lambda_{i,j}(t) = f(\mathbf{h}_i^t, \mathbf{h}_j^t)$$

# Temporal GNN layer

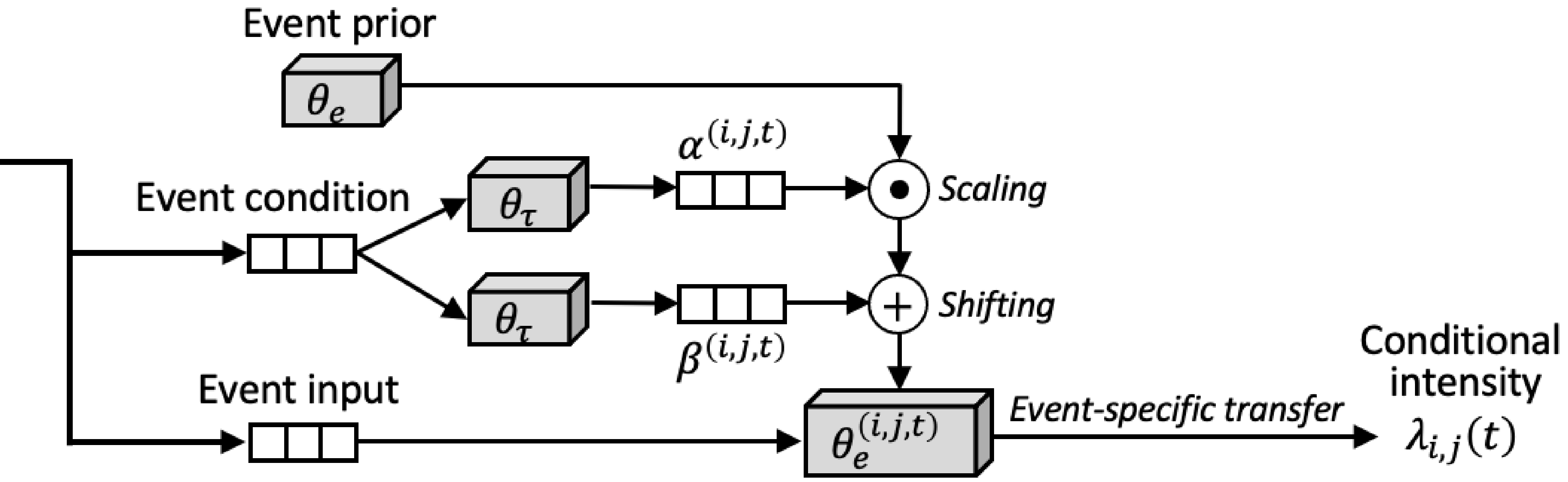



$$\frac{\kappa(t-t')}{\sum_{(i,j'',t'')\in\mathcal{H}_i(t)} \kappa(t-t'')}$$

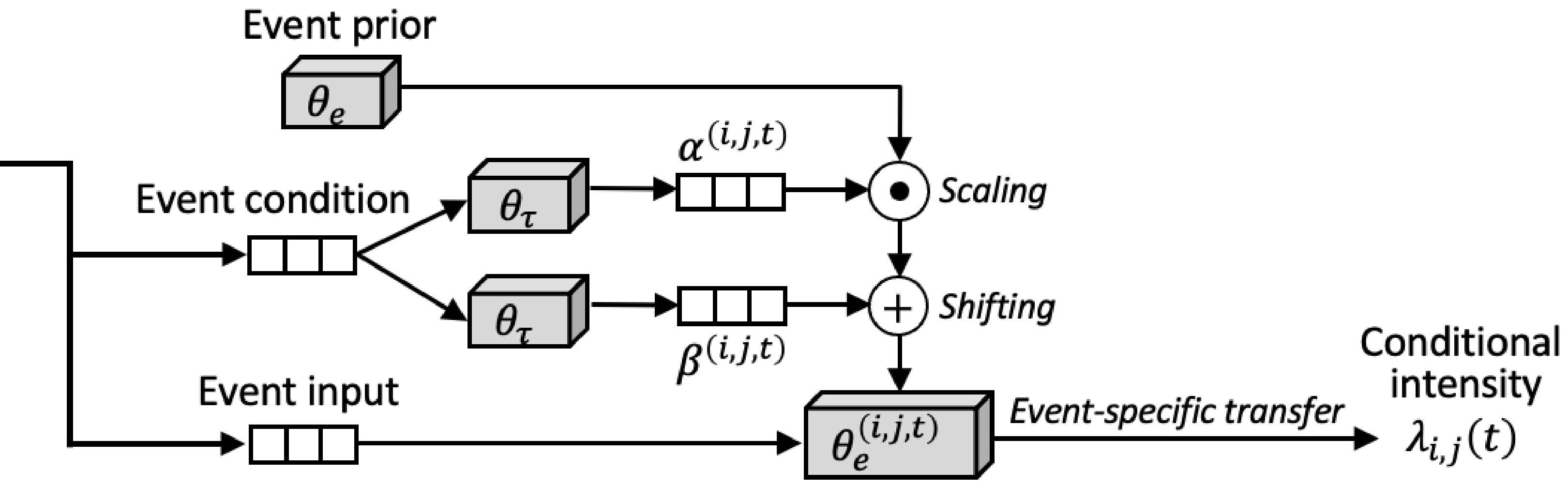
$$\mathbf{h}_i^{t,l} = \sigma\Big( \underbrace{\mathbf{h}_i^{t,l-1}\mathbf{W}^l_{\text{self}}}_{\substack{\text{self-information} \\ \text{(for base intensity)}}} + \underbrace{\sum_{(i,j',t')\in\mathcal{H}_i(t)} \mathbf{h}_{j'}^{t',l-1}\mathbf{W}^l_{\text{hist}}\tilde{\kappa}_i(t-t')}_{\substack{\text{historical neighbors' information} \\ \text{(for excitement by historical events)}}} \Big)$$

# Modeling event dynamics

$$\lambda_{i,j}(t) = f(\mathbf{h}_i^t, \mathbf{h}_j^t) = \mathrm{FCL}_e((\mathbf{h}_i^t - \mathbf{h}_j^t)^{\circ 2}; \theta_e)$$

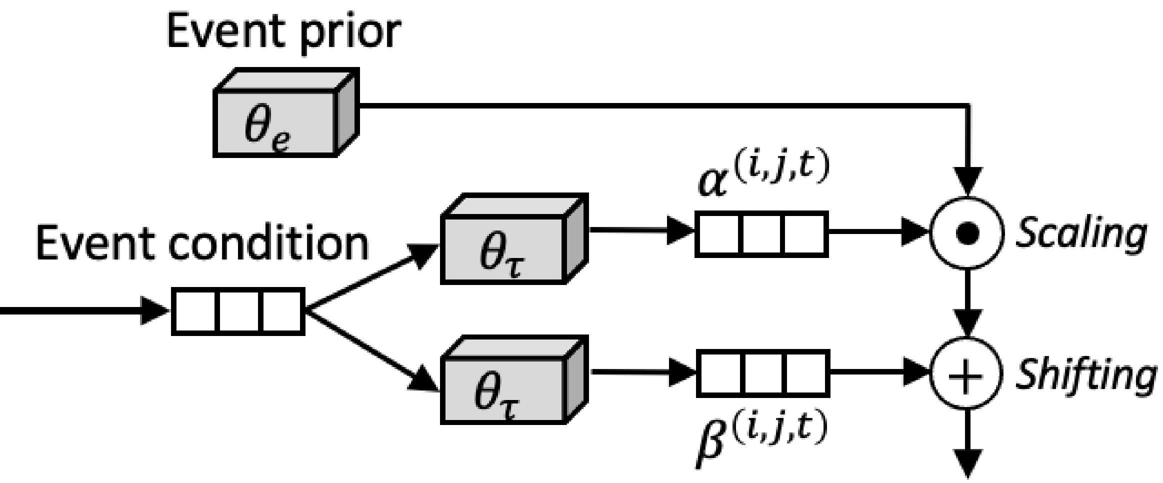


# Event prior and adaptation




$$\theta_e^{(i,j,t)} = \tau(\theta_e, \mathbf{h}_i^t \| \mathbf{h}_j^t; \theta_\tau) \qquad \lambda_{i,j}(t) = \mathrm{FCL_e}((\mathbf{h}_i^t - \mathbf{h}_j^t)^{\circ 2}; \theta_e^{(i,j,t)})$$

# Learnable transformation




a vector of ones

element-wise multiplication

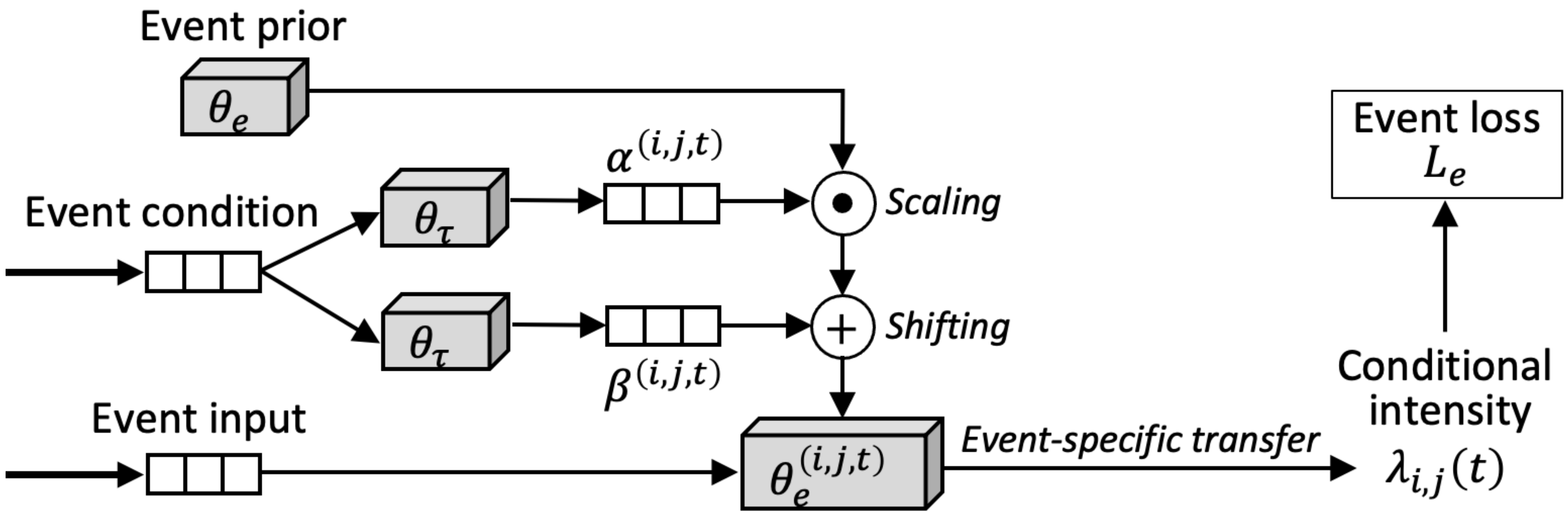
$$\theta_e^{(i,j,t)} = \tau(\theta_e, \mathbf{h}_i^t \| \mathbf{h}_j^t; \theta_\tau) = (\alpha^{(i,j,t)} + \mathbf{1}) \odot \theta_e + \beta^{(i,j,t)}$$

$$\alpha^{(i,j,t)} = \sigma\big((\mathbf{h}_i^t \| \mathbf{h}_j^t)\mathbf{W}_\alpha + \mathbf{b}_\alpha\big)$$

$$\beta^{(i,j,t)} = \sigma\big((\mathbf{h}_i^t \| \mathbf{h}_j^t)\mathbf{W}_\beta + \mathbf{b}_\beta\big)$$

# Event loss

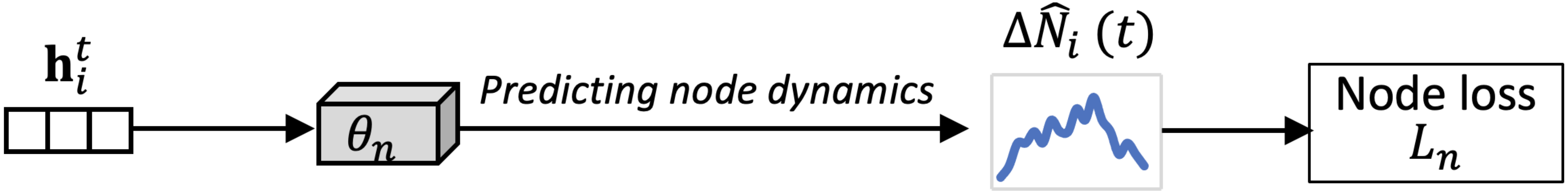
$$L_e(i, j, t) = -\log(\lambda_{i,j}(t)) - Q \cdot \mathbb{E}_{k\sim P_n} \log(1 - \lambda_{i,k}(t))$$



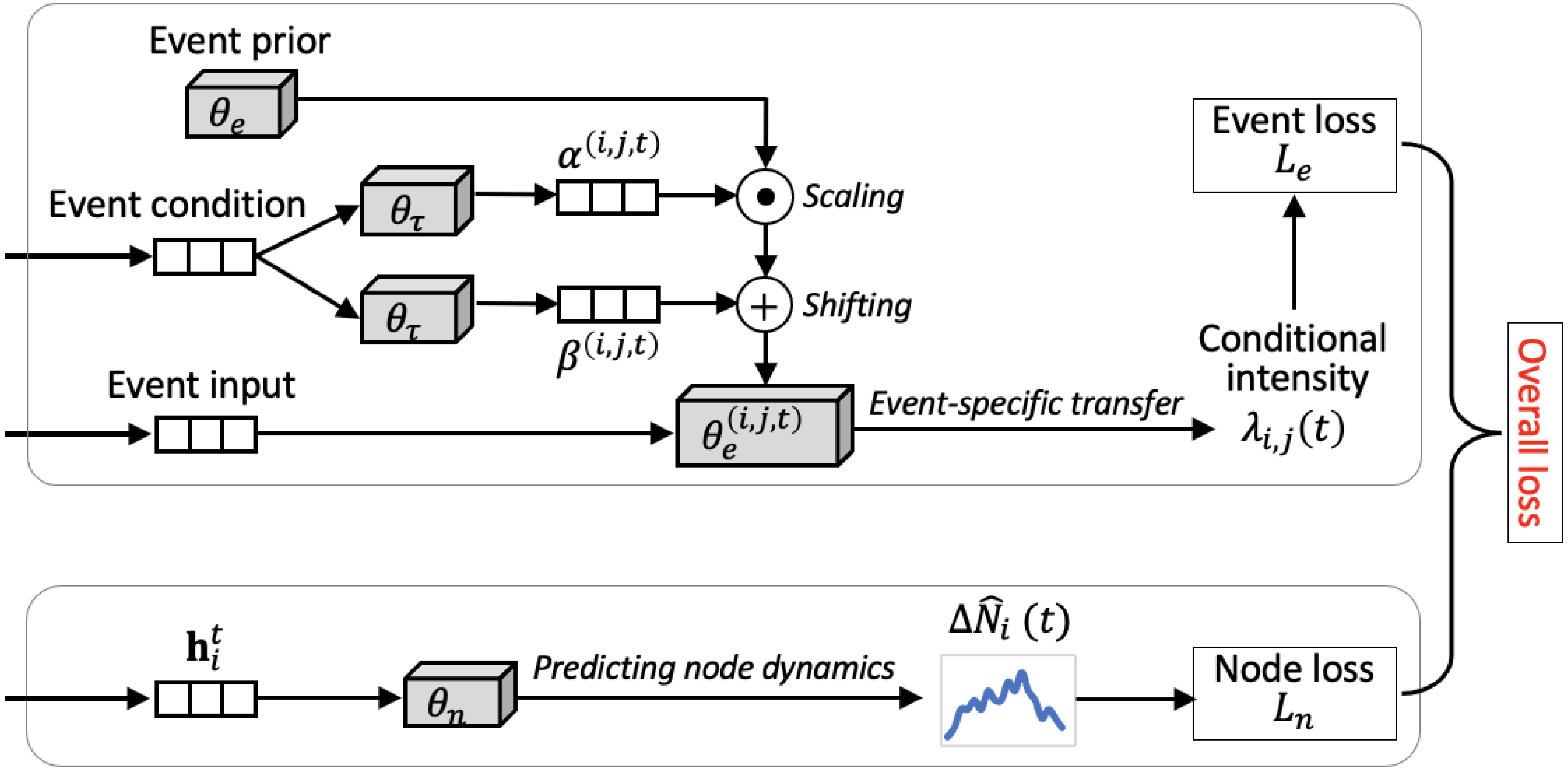
# Modeling node dynamics

Predicted number of new events occurring on the node at $t$

$$\Delta\hat{N}_i(t) = \mathrm{FCL}_n(\mathbf{h}_i^t; \theta_n)$$



# Overall loss




$$\arg\min_{\Theta} \sum_{(i,j,t)\in\mathcal{I}^{\text{tr}}} L_e + \eta_1 L_n + \eta_2(\|\alpha^{(i,j,t)}\|_2^2 + \|\beta^{(i,j,t)}\|_2^2)$$

$(\theta_g, \theta_e, \theta_\tau, \theta_n)$

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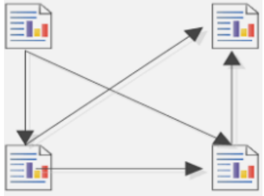
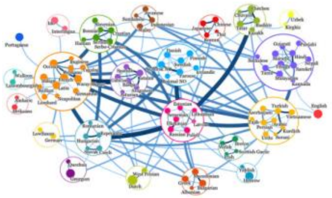
# Statistics of datasets

| Dataset | CollegeMsg | cit-HepTh | Wikipedia | Taobao |
|---|---|---|---|---|
| # Events | 59,835 | 51,315 | 157,474 | 4,294,000 |
| # Nodes | 1,899 | 7,577 | 8,227 | 1,818,851 |
| # Node features | – | 128 | 172 | 128 |
| Multi-edge? | Yes | No | Yes | Yes |
| New nodes in testing | 22.79% | 100% | 7.26% | 23.46% |

https://www.shutterstock.com/th/video/clip-21752794-message-network-icon-link-connection-technology-loop





https://www.researchgate.net/publication/297894915

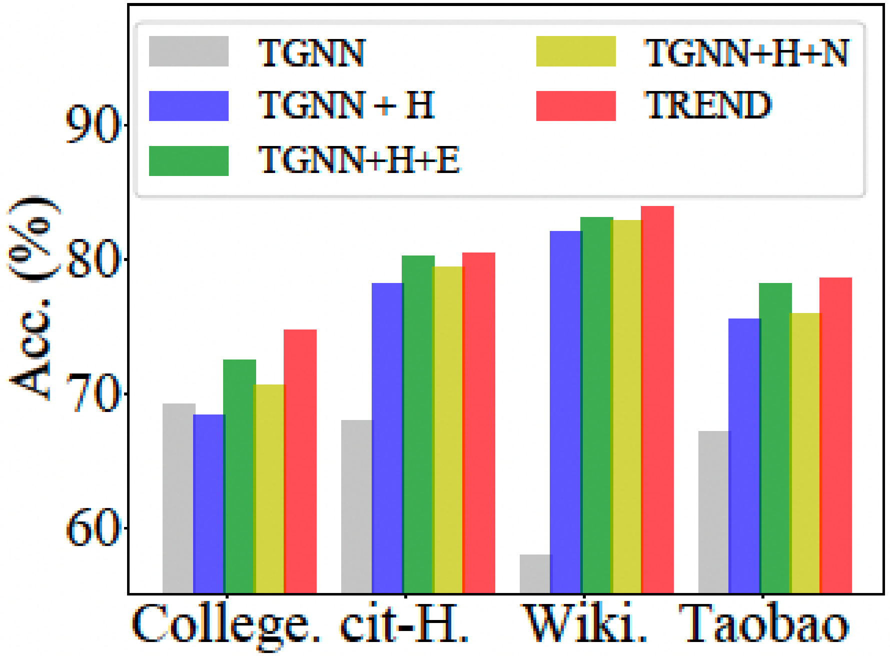
https://www.dreamstime.com/concept-e-commerce-shopping-web-icons-line-style-mobile-shop-digital-marketing-bank-card-gifts-digital-concept-e-commerce-image159818445

# Performance comparison with baselines

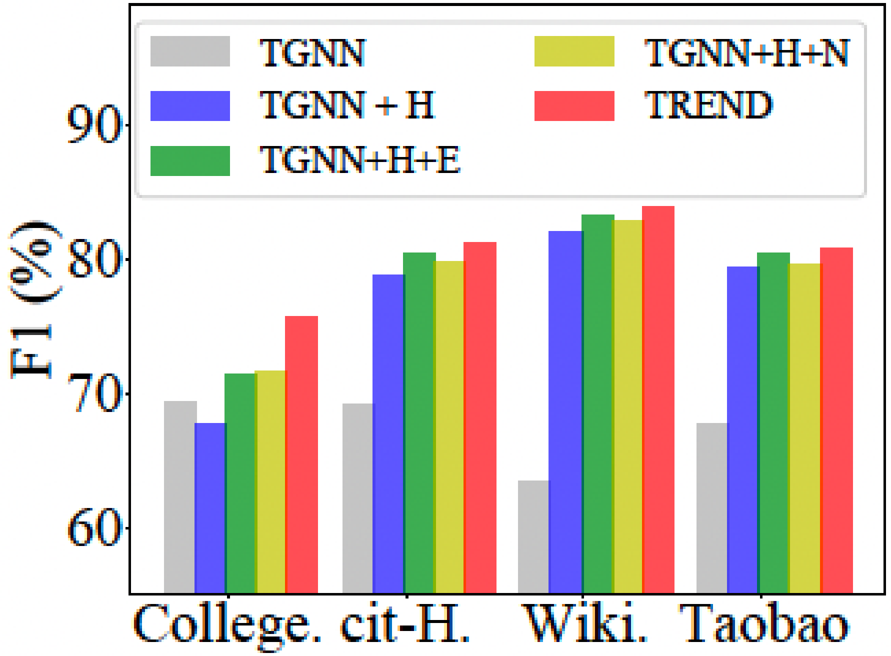
In each column, the best result is **bolded** and the runner-up is underlined. Improvement by TREND is calculated relative to the best baseline. "-" indicates no result obtained due to out of memory issue; $*$ indicates that our model significantly outperforms the best baseline based on two-tail $t$ -test ($p < 0.05$).

| | | CollegeMsg | | cit-HepTh | | Wikipedia | | Taobao | |
|---|---|---|---|---|---|---|---|---|---|
| | | Accuracy | F1 | Accuracy | F1 | Accuracy | F1 | Accuracy | F1 |
| Static | DeepWalk | 66.54±5.36 | 67.86±5.86 | 51.55±0.90 | 50.39±0.98 | 65.12±0.94 | 64.25±1.32 | 53.59±0.18 | 56.67±0.12 |
| | Node2vec | 65.82±4.12 | 69.10±3.50 | 65.68±1.90 | 66.13±2.15 | 75.52±0.58 | 75.61±0.52 | 52.74±0.33 | 54.86±0.32 |
| | VGAE | 65.82±5.68 | 68.73±4.49 | 66.79±2.58 | 67.27±2.84 | 66.35±1.48 | 68.04±1.18 | 55.97±0.22 | 59.80±0.16 |
| | GAE | 62.54±5.11 | 66.97±3.22 | 69.52±1.10 | 70.28±1.33 | 68.70±1.34 | 69.74±1.43 | 58.13±0.15 | 61.40±0.07 |
| | GraphSAGE | 58.91±3.67 | 60.45±4.22 | 70.72±1.96 | 71.27±2.41 | 72.32±1.25 | 73.39±1.25 | 60.74±0.18 | 61.61±0.20 |
| Temporal | CTDNE | 62.55±3.67 | 65.56±2.34 | 49.42±1.86 | 44.23±3.92 | 60.99±1.26 | 62.71±1.49 | 51.64±0.32 | 43.99±0.38 |
| | EvolveGCN | 63.27±4.42 | 65.44±4.72 | 61.57±1.53 | 62.42±1.54 | 71.20±0.88 | 73.43±0.51 | - | - |
| | GraphSAGE+T | 69.09±4.91 | 69.41±5.45 | 67.80±1.27 | 69.12±1.12 | 57.93±0.53 | 63.41±0.91 | 67.05±0.23 | 67.69±0.17 |
| | TGAT | 58.18±4.78 | 57.23±7.57 | 78.02±1.93 | 78.52±1.61 | 76.45±0.91 | 76.99±1.16 | 70.07±0.59 | 71.31±0.18 |
| Hawkes process | HTNE | 73.82±5.36 | 74.24±5.36 | 66.70±1.80 | 67.47±1.16 | 77.88±1.56 | 78.09±1.40 | 59.03±0.17 | 60.34±0.19 |
| | MMDNE | 73.82±5.36 | 74.10±3.70 | 66.28±3.87 | 66.70±3.39 | 79.76±0.89 | 79.87±0.95 | 58.24±0.10 | 59.04±0.16 |
| | TREND (improv.) | **74.55**±1.95 (+0.99%) | **75.64**±2.09 (+1.89%) | **80.37**$^*$±2.08 (+3.01%) | **81.13**$^*$±1.92 (+3.32%) | **83.75**$^*$±1.19 (+5.00%) | **83.86**$^*$±1.24 (+4.99%) | **78.56**$^*$±0.17 (+12.11%) | **80.67**$^*$±0.15 (+13.12%) |

# Ablation study




(a) Accuracy




(b) F1

# Outline

# Conclusion

- Conclusion:
  - Studied the problem of temporal graph representation learning and temporal link prediction.
  - Proposed TREND, a novel framework driven by event and node dynamics on a Hawkes process-based GNN.
  - Conduct extensive experiments on four real-world graph datasets and demonstrated the superior performance of TREND.

# THANK YOU FOR YOUR ATTENTION

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