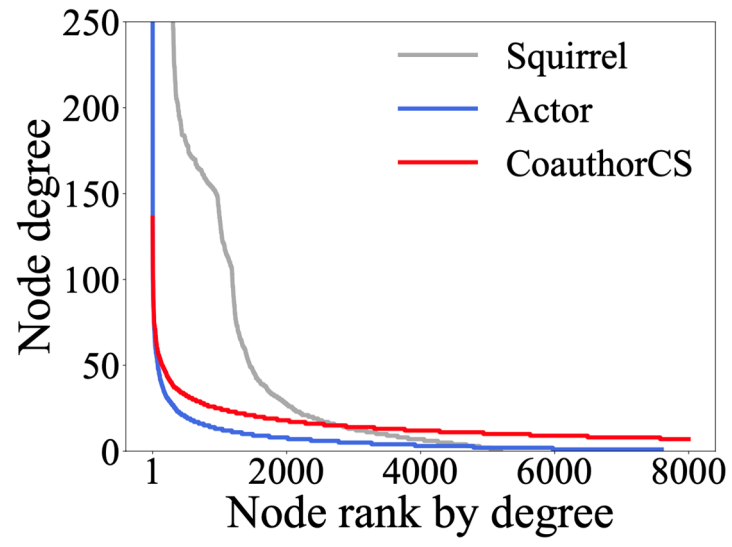
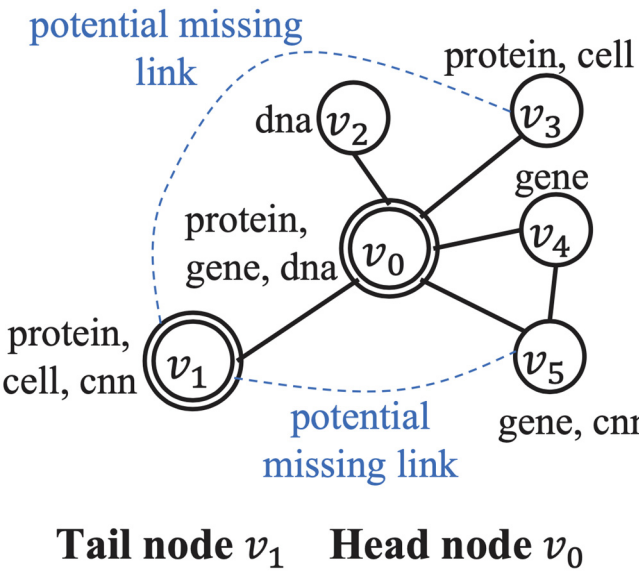


## Motivation



(a) Long-tailed node distribution



(b) Toy citation network

**Problem:** Robust tail node embedding.

### Tail nodes

- Small neighborhood
- Potentially suffer from missing information

### Challenges:

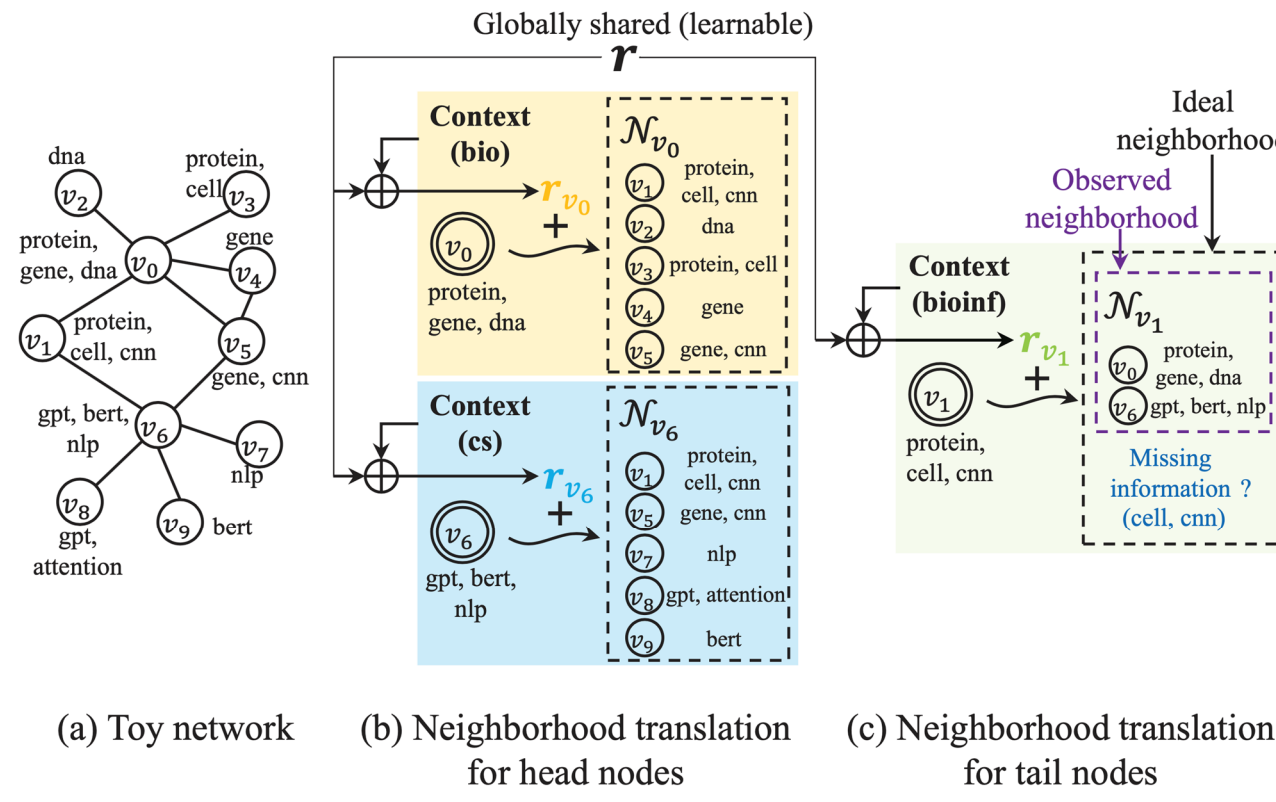
*C1: How to uncover the missing neighborhood information for tail nodes?*

*C2: How to localize the missing information for each tail node while maintaining the generality across nodes?*

## The proposed model: Tail-GNN

### Neighborhood translation

$$\mathbf{h}_v + \mathbf{r}_v \approx \mathbf{h}_{\mathcal{N}_v}$$



### Key idea

- Neighborhood translation

### First challenge

- predict the missing neighborhood information for tail nodes by exploiting a transferable neighborhood translation

### Second challenge

- tailor the shared neighborhood translation to each target node w.r.t. its local context.

### Neighborhood of head nodes

- Observed neighborhood: complete and representative
- no missing information

$$\mathbf{m}_v = \mathbf{h}_{\mathcal{N}_v^*} - \mathbf{h}_{\mathcal{N}_v} = 0$$

### Neighborhood of tail nodes

- Observed neighborhood: not representative enough
- Imperative: uncover the missing information

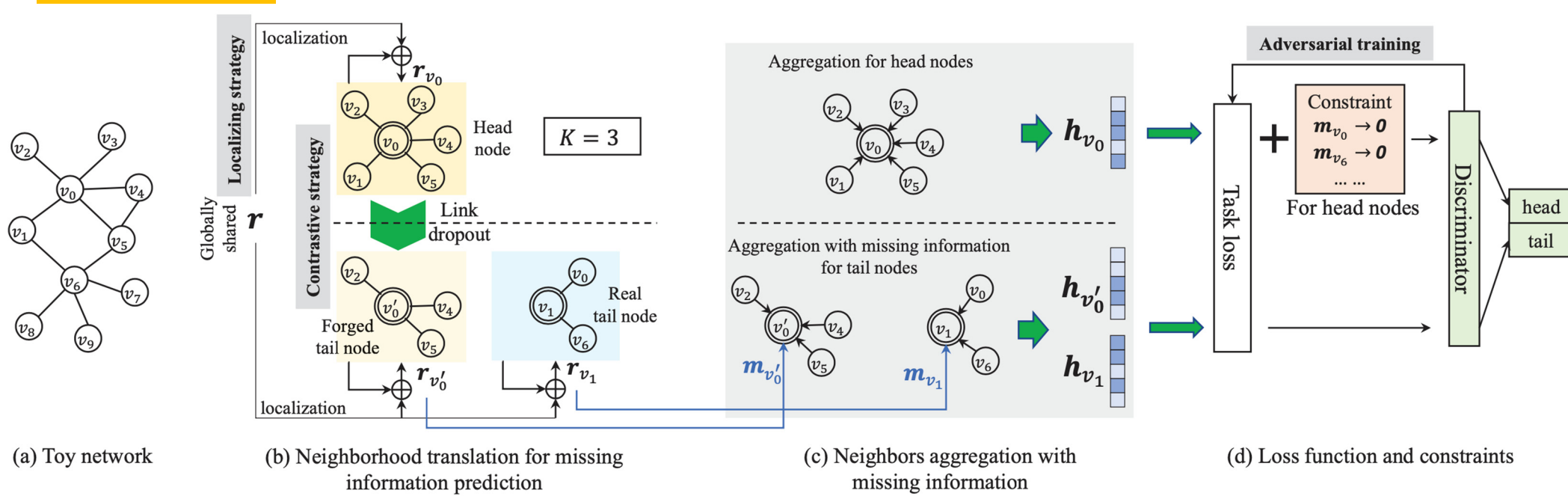
$$\mathbf{m}_v = \mathbf{h}_{\mathcal{N}_v^*} - \mathbf{h}_{\mathcal{N}_v} \neq 0$$

Predicting missing information for tail node  $v$

$$\mathbf{h}_{\mathcal{N}_v^*} = \mathbf{h}_v + \mathbf{r}_v$$

$$\mathbf{m}_v = \mathbf{h}_v + \mathbf{r}_v - \mathbf{h}_{\mathcal{N}_v}$$

### Tail-GNN



### Contrastive strategy

#### Head nodes

$$\mathbf{h}_v^l + \mathbf{r}_v^l - \mathbf{h}_{\mathcal{N}_v}^l \rightarrow 0$$

#### Tail nodes

$$\mathbf{m}_v^l = \mathbf{h}_{\mathcal{N}_v^*}^l - \mathbf{h}_{\mathcal{N}_v}^l = \mathbf{h}_v^l + \mathbf{r}_v^l - \mathbf{h}_{\mathcal{N}_v}^l$$

### Localizing strategy

$$\mathbf{r}_v^l = \phi(\mathbf{h}_v^l, \mathbf{h}_{\mathcal{N}_v}^l, \mathbf{r}^l; \theta_\phi^l) = (\gamma_v^l + 1) \odot \mathbf{r}^l + \beta_v^l$$

### Neighborhood aggregation

#### Head nodes

$$\mathbf{h}_v^{l+1} = \mathcal{M}(\mathbf{h}_v^l, \{\mathbf{h}_i^l : i \in \mathcal{N}_v\}; \theta^{l+1})$$

Message passing function

#### Tail nodes

$$\mathbf{h}_v^{l+1} = \mathcal{M}(\mathbf{h}_v^l, \{\mathbf{m}_i^l\} \cup \{\mathbf{h}_i^l : i \in \mathcal{N}_v\}; \theta^{l+1})$$

missing neighborhood information observed neighborhood

### Overall objective

$$\text{Task loss } \mathcal{L}_t = \sum_{v \in \mathcal{V}_{tr}} \text{CROSSENT}(\mathbf{h}_v^l, y_v) + \lambda_t \|\Theta\|_2^2$$

### Overall objective

$$\text{Loss for head missing information constraint } \mathcal{L}_m = \sum_{v \in \mathcal{V}_{tr}} I_v \sum_{l=1}^L \|\mathbf{m}_v^{l-1}\|_2^2$$

$$\min_{\Theta} \max_{\theta_d} \mathcal{L}_t + \mu \mathcal{L}_m - \eta \mathcal{L}_d$$

$$\text{Loss for adversarial constraint } \mathcal{L}_d = \sum_{v \in \mathcal{V}_{tr}} \text{CROSSENT}(I_v, D(\mathbf{h}_v^l; \theta_d)) + \lambda_d \|\theta_d\|_2^2$$

Discriminator

## Experiments

### Experimental setup

	# Nodes	# Edges	# Features	# Classes	# Tail ( $K = 5$ )
Email	1,005	25,571	128	42	235
Squirrel	5,201	217,073	2,089	5	942
Actor	7,600	33,391	931	5	4,823
CoauthorCS	18,333	327,576	6,805	15	8,037
Amazon	937,349	12,455,925	100	44	248,125

#### Base GNN models

- GCN [1]
- GAT [2]
- GraphSAGE [3]

#### Baselines (based on GCN)

- Conventional: DeepWalk [4], GCN [1]
- Refinement: Additive [5], a la carte [6], meta-tail2vec [7]
- Robust models: SDNE [8], ARG [9], DDGCN
- Degree-aware models: Demo-Net [11], role2vec

### Node classification

**Table 2: Evaluation on tail node classification using GCN as the base model.**

Henceforth, tabular results are in percent; the best result is **bolded** and the runner-up is underlined; a dash (-) denotes no result reported for failing to work on a large dataset.

Methods	Email Accuracy	Email Micro-F	Squirrel Accuracy	Squirrel Micro-F	Actor Accuracy	Actor Micro-F	CoauthorCS Accuracy	CoauthorCS Micro-F	Amazon Accuracy	Amazon Micro-F
DeepWalk	54.4 ± 0.3	51.3 ± 0.3	<b>28.8 ± 1.6</b>	<b>28.0 ± 2.3</b>	21.8 ± 0.6	18.2 ± 0.9	84.1 ± 0.7	81.5 ± 0.7	83.7 ± 0.1	<b>74.3 ± 0.6</b>
GCN	<u>57.9 ± 1.2</u>	<u>57.7 ± 1.3</u>	24.8 ± 1.3	23.2 ± 1.8	<u>29.7 ± 0.2</u>	15.0 ± 0.9	88.4 ± 0.1	86.1 ± 0.1	82.3 ± 0.2	70.6 ± 0.1
Additive	55.4 ± 0.4	52.5 ± 0.2	27.0 ± 1.7	22.9 ± 1.6	28.1 ± 0.3	15.1 ± 1.3	89.5 ± 0.1	87.8 ± 0.1	84.2 ± 0.2	73.2 ± 0.6
a la carte	21.1 ± 0.4	17.9 ± 0.5	22.5 ± 1.1	22.5 ± 0.7	28.0 ± 0.5	14.8 ± 1.4	88.7 ± 0.2	86.7 ± 0.3	81.1 ± 0.1	69.7 ± 0.7
meta-tail2vec	57.1 ± 0.1	55.3 ± 0.2	25.1 ± 0.5	21.5 ± 0.3	<u>29.7 ± 0.4</u>	20.1 ± 0.7	89.3 ± 0.1	87.4 ± 0.1	81.9 ± 0.1	71.4 ± 0.4
SDNE	32.9 ± 0.6	29.8 ± 0.5	23.8 ± 3.2	16.6 ± 6.2	24.4 ± 0.8	12.6 ± 5.6	70.6 ± 0.9	64.5 ± 1.1	-	-
ARGA	45.1 ± 0.9	41.2 ± 1.0	22.4 ± 1.0	22.8 ± 1.9	25.9 ± 0.3	8.2 ± 0.6	74.6 ± 1.8	67.9 ± 2.5	-	-
DDGCN	39.8 ± 0.6	38.9 ± 0.7	26.3 ± 2.1	26.4 ± 3.3	24.0 ± 0.4	11.7 ± 0.7	73.6 ± 0.9	68.8 ± 1.0	-	-
DEMO-Net	56.9 ± 0.6	56.5 ± 0.7	28.3 ± 0.5	22.5 ± 2.2	28.4 ± 0.8	22.0 ± 1.3	90.8 ± 0.5	88.9 ± 0.6	83.1 ± 0.1	72.0 ± 0.4
role2vec	44.9 ± 1.6	43.8 ± 2.4	26.3 ± 0.8	27.5 ± 1.7	23.1 ± 0.1	18.3 ± 0.6	62.7 ± 0.3	56.3 ± 0.3	77.1 ± 0.2	61.5 ± 0.5
Tail-GCN	59.2 ± 0.8	<b>58.5 ± 1.3</b>	<b>30.2 ± 1.1</b>	<b>31.1 ± 1.1</b>	34.9 ± 0.5	25.2 ± 0.6	93.6 ± 0.1	92.7 ± 0.1	<b>87.0 ± 0.1</b>	<b>78.2 ± 0.2</b>

**Table 3: Evaluation on tail node classification using other GNNs as the base model.**

Methods	Email Accuracy	Email Micro-F	Squirrel Accuracy	Squirrel Micro-F	Actor Accuracy	Actor Micro-F	CoauthorCS Accuracy	CoauthorCS Micro-F	Amazon Accuracy	Amazon Micro-F
GAT	57.9 ± 0.4	57.3 ± 0.2	24.1 ± 2.4	23.1 ± 2.6	29.8 ± 0.6	13.2 ± 2.7	88.6 ± 0.2	86.2 ± 0.2	-	-
Tail-GAT	<b>59.4 ± 0.9</b>	<b>58.2 ± 1.2</b>	<b>28.8 ± 2.1</b>	<b>30.4 ± 2.6</b>	<b>34.5 ± 1.3</b>	<b>24.7 ± 2.0</b>	<b>92.5 ± 0.1</b>	<b>90.8 ± 0.1</b>	-	-
GraphSAGE	52.0 ± 1.6	51.3 ± 1.7	27.1 ± 2.7	26.4 ± 4.9	33.1 ± 1.1	23.2 ± 2.4	89.8 ± 2.4	87.7 ± 1.1	79.1 ± 0.4	62.8 ± 0.6
Tail-GraphSAGE	55.7 ± 0.6	54.9 ± 0.7	28.5 ± 1.6	28.2 ± 2.4	34.1 ± 1.7	26.8 ± 1.8	93.8 ± 0.7	92.4 ± 1.4	<b>85.1 ± 0.2</b>	<b>75.5 ± 0.3</b>

#### GCN as base model

- **DEMO-Net, role2vec**: can distinguish nodes of different degrees, not specifically for enhancing the tail nodes.
- **SDNE, ARG and DDGCN**: improve the robustness of graph learning, not specifically target the tail nodes.
- **Refinement models**: in two stages, the embedding stage cannot benefit from the refinement stage.

#### GAT and GraphSAGE as base models

- Tail-GNN still outperforms the baselines, showing its flexibility.

### Ablation study

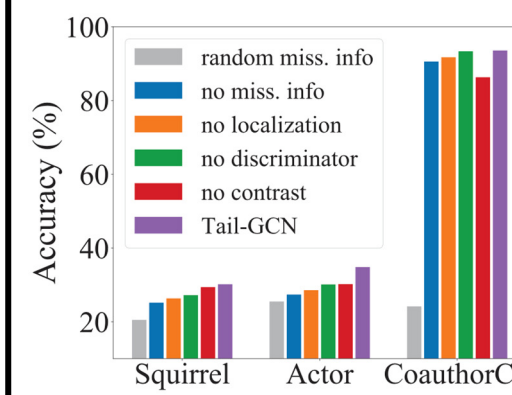


Figure 4: Ablation study.

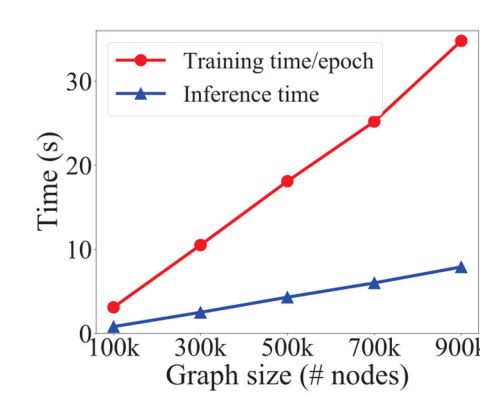


Figure 5: Scalability study.

#### Ablation study

- Random/no missing info: impairs the performance
- Without localization: hurts the performance
- Discriminator: contributes to the performance
- Without contrastive strategy: performance becomes worse

#### Scalability

- Increase linearly w.r.t. graph size

## Conclusions

### Problem

- **Tail node embedding** in graph neural networks

### Proposed model: Tail-GNN

- A new concept of **transferable neighborhood translation**
  - to capture the **relational tie** between a node and its neighboring nodes
- A novel model **Tail-GNN**
  - to narrow the gap between head and tail nodes for robust tail node embedding

### Experiments

- Extensive experiments demonstrate that Tail-GNN significantly outperforms state-of-the-art models.

## Reference

- [1] Kipf, T. N., et al. 2017. Semi-supervised classification with graph convolutional networks. ICLR.
- [2] Veličković, P., et al. 2018. Graph attention networks. ICLR.
- [3] Hamilton W L., et al. 2017. Inductive representation learning on large graphs. NeurIPS.
- [4] Perozzi B., et al. 2014. Deepwalk: Online learning of social representations. KDD.
- [5] Lazaridou A, et al. 2017. Multimodal word meaning induction from minimal exposure to natural text. Cognitive science.
- [6] Khodak M, et al. 2018. A la carte embedding: Cheap but effective induction of semantic feature vectors. ACL.
- [7] Liu Z, et al. 2020. Towards locality-aware meta-learning of tail node embeddings on networks. CIKM.
- [8] Wang D, et al. 2016. Structural deep network embedding. KDD.
- [9] Pan S, et al. 2018. Adversarially regularized graph autoencoder for graph embedding. IJCAI.
- [10] Cai R, et al. 2020. Dual-dropout graph convolutional network for predicting synthetic lethality in human cancers. Bioinformatics.
- [11] Wu J, et al. 2019. Demo-Net: Degree-specific graph neural networks for node and graph classification. KDD.
- [12] Ahmed N, et al. 2020. Role-based graph embeddings. TKDE.