End-to-End Open-Set Semi-Supervised Node Classification with Out-of-Distribution Detection

Tiancheng Huang^{1,2,3}, Donglin Wang^{2,3*}, Yuan Fang⁴, and Zhengyu Chen^{2,3} 1 Zhejiang University, Hangzhou, China 2 Westlake University, Hangzhou, China 3 Westlake Institute for Advanced Study, Hangzhou, China 4 Singapore Management University, Singapore

{huangtiancheng, wangdonglin, chenzhengyu}@westlake.edu.cn, yfang@smu.edu.sg

Motivation

OOD Detection on Images [1]



W/O Propagation and Aggregation between ID and OOD Samples

OOD Detection on Graphs (Ours)

Across-distribution Mixture



W/ Propagation and Aggregation between ID and OOD Samples

[1] Yu Q, Aizawa K. Unsupervised Out-of-Distribution Detection by Maximum Classifier Discrepancy. ICCV 2019

Across-distribution Mixture

Theorem. Across-distribution Mixture [1]. It is assumed that the sample feature conforms to the Normal distribution with the mean μ and variance σ . The mixing feature of a sample comes from in-distribution and out-of-distribution:

$$P_{mix}(\mathbf{x}) = P(\mathbf{x}|O = in)P(O = in) + P(\mathbf{x}|O = out)P(O = out)$$

 $\sim \mathcal{N}(\mu_1, \sigma_1) + \mathcal{N}(\mu_2, \sigma_2),$

where $P(\mathbf{x}|O=in), P(\mathbf{x}|O=out) \sim \mathcal{N}(\mu_i, \sigma_i), i \in \{1, 2\}.$

Lemma. *Across-distribution Mixture on Graphs.* Take one-layer aggregation of GNN as an example, the distribution mixture comes from the central node and its neighbors:

$$P_{mix}(\mathbf{x}_i) \sim \mathcal{N}(\mu_i, \sigma_i) + \sum_{v=1}^{|\mathbb{N}(u)|} w_{v,u} \mathcal{N}(\mu_{j_v}, \sigma_{j_v})$$

where $i, j_v \in \{1, 2\}, w_{v,u}$ denotes weights between node u and v, and $\mathbb{N}(u)$ denotes neighbors of node u

[1] Bitterwolf et al. Revisiting ood detection: A simple baseline is surprisingly effective. ICLR 2022 Submitted.

Unified Learning Framework

To avoid the across-distribution mixture

The joint probability distribution of node label Y and latent variable O

 $P_{\theta}(Y, O | \mathbf{X}, \mathbf{A}) = P_{\theta}(Y | \mathbf{X}, \mathbf{A}, O) P(O | \mathbf{X}, \mathbf{A}),$

i) Learning the GNN parameter by maximizing the likelihood

$$\log P_{\theta}(Y, O | \mathbf{X}, \mathbf{A}) = \log \sum_{k} P_{\theta}(Y | \mathbf{X}, \mathbf{A}, O_{k}) \cdot P(O_{k} | \mathbf{X}, \mathbf{A}),$$

ii) Inferring the following posterior of the latent variable O as

$$P_{\theta}(O_k | \mathbf{X}, \mathbf{A}, Y) = \frac{P_{\theta}(Y | \mathbf{X}, \mathbf{A}, O_k)}{\sum_j P_{\theta}(Y | \mathbf{X}, \mathbf{A}, O_j)},$$

Challenges

1) involves marginalizing the latent variable O

2) lacks of supervision for test nodes for inference

Unified Learning Framework

Variational Inference

Introducing variational distribution Q

 $\mathcal{L}(\theta) = \mathbb{E}_{Q(O_k)} \left[\log P_{\theta}(Y | \mathbf{X}, \mathbf{A}, O_k) \right] - \mathrm{KL}(Q(O_k) || P(O_k)),$

Introducing the parameterized posterior Q_{ϕ} with parameter ϕ , and minimizing Kullback-Leibler (KL) divergence, to make the variational distribution Q_{ϕ} close to its intractable true posterior distribution

$$\mathrm{KL}(Q_{\phi} \| P) = \mathbb{E}_{Q} \left[\log \frac{Q_{\phi}(O_{k} | \mathbf{X}, \mathbf{A})}{P(O_{k})} \right]$$

where P follows Bernoulli distribution

Negative ELBO $\mathcal{L}(\theta, \phi) = -\mathbb{E}_{Q_{\phi}}[\log P_{\theta}(Y|\mathbf{X}, \mathbf{A}, O)] + \mathrm{KL}(Q_{\phi} \| P),$

Learning to Mix Neighbors

Learning and Aggregating Weights

Predictor f_{ϕ} gives a single scalar between 0 and 1 (parametrized as a sigmoid):

$$Q_{\phi}(w_{v,u}|\mathbf{X}, \mathbf{A}) = \frac{1}{1 + exp(-\mathbf{W}^{T}[\mathbf{H}_{v} \| \mathbf{H}_{u}])},$$



Bi-level Optimization

Updating outer level

Updating inner level

$$\min_{\phi} \mathcal{L}_{val}(\theta^*(\phi), \phi),$$

s.t. $\theta^*(\phi) = \arg\min_{\theta} \mathcal{L}_{train}(\theta, \phi),$

Experiments

Datasets. 1) Cora; 2) Citeseer; 3) Pubmed; and 4) ogbn-arXiv

For the split of OOD classes, we strictly follow the standard OOD detection benchmark on graphs [Stadler et al., 2021]. The statistics of datasets are presented in the Table

below.

| | Cora | Citeseer | Pubmed | arXiv |
|--------------------------|--------|----------|--------|-----------|
| # Nodes | 2,708 | 3,327 | 19,717 | 169,343 |
| # Edges | 10,556 | 9,104 | 88,648 | 2,315,598 |
| # Features | 1,433 | 3,703 | 500 | 128 |
| # Labels | 7 | 6 | 3 | 40 |
| $\# \mathcal{C}_{out} $ | 3 | 2 | 1 | 15 |
| # Fraction | 33.38% | 33.18% | 39.94% | 39.11% |

Baselines.

GCN [Kipf and Welling, 2017], 2) ChebNet [Defferrard et al., 2016], 3) GraphSAGE [Hamilton et al., 2017], 4) GAT [Veli^{*}ckovi^{*} c et al., 2018], 5) SGC [Wu et al., 2019], 6)
JKNet [Xu et al., 2018], 7) APPNP [Klicpera et al., 2018], 8) SuperGAT [Kim and Oh, 2020],
9) GCNII [Chen et al., 2020], and 10) DropEdge [Rong et al., 2019].

Experiments

Comparison of **semi-supervised node classification** accuracy (%)

Comparison of **OOD detection** AUROC (%)

| Methods | Cora | Citeseer | Pubmed | arXiv | Methods | Cora | Citeseer | Pubmed | arXiv |
|-----------|----------------------------|----------------------------|------------------------------|---|-----------|------------------|------------------------------|----------------------------|------------------------------|
| GCN | 87.4 ± 0.3 | 66.0 ± 0.6 | 89.0 ± 0.2 | $47.4{\scriptstyle\pm0.6}$ | GCN | 77.8±0.4 | $73.1_{\pm 2.2}$ | 63.3 ± 1.4 | 56.1 ± 0.5 |
| ChebNet | 85.6 ± 0.4 | 65.0 ± 0.6 | 88.4 ± 0.3 | $46.5{\scriptstyle \pm 0.4}$ | ChebNet | 73.5±1.3 | $69.7_{\pm 4.0}$ | 62.2 ± 1.2 | $57.1{\scriptstyle \pm 0.8}$ |
| GraphSAGE | 85.3 ± 1.2 | 65.8 ± 0.7 | 89.6 ± 0.6 | 46.8 ± 0.9 | GraphSAGE | 75.6±1.8 | 72.8 ± 3.1 | $59.5{\scriptstyle\pm2.0}$ | $56.9{\scriptstyle \pm 1.0}$ |
| GAT | $88.7{\scriptstyle\pm0.6}$ | 69.6 ± 0.6 | 90.6 ± 0.9 | $49.8{\scriptstyle \pm 1.5}$ | GAT | 80.2 ± 1.4 | $77.9_{\pm 3.1}$ | 61.6±4.2 | $58.0{\scriptstyle \pm 1.0}$ |
| SGC | 87.2 ± 0.3 | 69.2 ± 0.2 | $91.5{\scriptstyle\pm0.6}$ | $40.5{\scriptstyle \pm 2.6}$ | SGC | $70.0_{\pm 0.8}$ | 75.5 ± 2.3 | 61.4 ± 1.8 | $51.8{\scriptstyle\pm1.5}$ |
| JKNet | 86.7 ± 1.1 | 67.3 ± 0.7 | $93.1{\scriptstyle\pm0.1}$ | 50.6 ± 0.6 | JKNet | 76.3±1.8 | 70.8 ± 3.4 | 64.4 ± 1.8 | $52.9{\scriptstyle\pm0.6}$ |
| APPNP | 88.2 ± 0.4 | 68.3 ± 0.5 | $93.2{\scriptstyle\pm0.1}$ | 51.3 ± 0.9 | APPNP | 77.8 ± 0.5 | 72.3 ± 2.7 | 64.3 ± 0.8 | $53.7{\scriptstyle\pm0.3}$ |
| SuperGAT | 88.3 ± 0.5 | $69.3{\scriptstyle\pm0.8}$ | $91.3{\scriptstyle\pm1.0}$ | $49.2{\scriptstyle\pm0.7}$ | SuperGAT | 78.5 ± 1.6 | 78.1 ± 1.6 | 63.2 ± 3.9 | $54.1{\scriptstyle \pm 0.8}$ |
| GCNII | 88.7 ± 0.3 | 69.4 ± 1.4 | $93.0{\scriptstyle\pm0.7}$ | 51.6 ± 1.7 | GCNII | 78.0±1.3 | $72.4_{\pm 2.1}$ | 65.2 ± 3.9 | $56.3{\scriptstyle \pm 2.2}$ |
| DropEdge | 88.9±0.7 | 69.6 ± 1.2 | $93.0{\scriptstyle \pm 0.9}$ | $51.7{\scriptstyle\pm2.7}$ | DropEdge | 79.3±0.9 | 75.2 ± 3.5 | 63.0 ± 2.1 | 57.9 ± 0.9 |
| LMN(Ours) | 89.7±0.6 | $71.1{\scriptstyle\pm0.6}$ | 93.4 ± 0.1 | $5\overline{4.1}{\scriptstyle \pm 1.4}$ | LMN(Ours) | 80.5±1.2 | $78.5{\scriptstyle \pm 3.2}$ | 68.7±1.3 | 60.4 ± 0.3 |

Experiments

1. Mixing Strategies

- 1) RandomMask (RM) 2) TruthMask (TM)
- 3) RandomDrop (RD) 4) TruthDrop (TD)

5) ATtention (AT)

6) LMN (Ours)



2. The Effect of Bi-level Optimization



Figure 4: The training and validation losses on Cora.

3. Ablation Study

| Methods | OOD Modules | Cora | Citeseer |
|---------|------------------|----------|----------------|
| GCNII | None | 88.7±0.3 | 69.4 ± 1.4 |
| LMN | Mixing Neighbors | 89.7±0.6 | 71.1 ± 0.6 |

Conclusions

- In this paper, we study a novel problem of end-to-end open-set semisupervised node classification with OOD detection.
- The novel method LMN in a variational inference framework has been proposed for node classification and OOD detection in an end-to-end manner.
- Extensive experiments on four datasets demonstrate the effectiveness of our proposed method.